Debt Sustainability Fan Charts: combining multivariate regression analysis and external forecasts [*]

Francisco Arizala, Christian Castro, Eduardo Cavallo[†] and Andrew Powell

Inter-American Development Bank

Abstract

We discuss the basics elements involved in the use of Fan Charts methodology applied to the Debt Sustainability Analysis (DSA) and we propose several approaches to integrate econometric-based techniques with external forecasts. In doing so, we propose techniques that create bridges between the deterministic and the uncertainty-based approaches for DSA. We also enumerate some pragmatic issues that should be considered in the process of designing a methodology for DSA's using fan charts.

Keywords: Debt Sustainability Analysis; Fan Charts.

JEL Codes: H63; H68

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[†] Corresponding author: E. Cavallo (cavalloe@iadb.org). 1300 New York Ave, N.W. Washington D.C., 20577, USA. Tel.: 1-202-623-2817; Fax.: 1-202-623-2481.

1. Introduction

Traditional debt sustainability analysis (DSA) frameworks rely on medium term simulations of the debt-to-GDP ratio given specific macroeconomic forecasts and fiscal policy assumptions. A key issue with the standard DSA's is the omission of uncertainty, for example about income growth, interest rates, fiscal policies, and exchange rates and other variables that may determine debt dynamics.

The framework of "Fan Charts" for DSA, rather than simply projecting one central scenario for the debt-to-GDP ratio, incorporates the structure of random shocks hitting the domestic economy to obtain a complete distribution of probable outcomes. This approach recognizes that even when the government is resolute in pursuing its fiscal targets, the outcomes are subject to significant risks, especially as the planning horizon lengthens.

In principle assessing debt sustainability involves two things: (1) contemplating the probable future debt paths given the existing projections about the fiscal policy stance and the expected evolution of the main macroeconomic variables; and (2) assessing the country's risk profile given its history and also the pattern of shocks that typically beset the country. In our view, the most common DSA approaches deal with either (1) or (2), but few attempts have been made to this date to combine the two approaches.[3] Thus, situations arise where for example the baseline projections for a given debt trajectory are subject to “stress tests” based upon changing one parameter but keeping the others fixed. Yet, this approach usually ignores, for example, that during periods of crises, developing economies experience co-movements in their macro variables that actually deteriorate debt sustainability: i.e., recession, fall in revenues, worst fiscal results, hike in interest rates, inflation and exchange rate depreciation.

On the other hand, DSAs that are based on purely econometric analyses of the historical co-movement of macroeconomic variables and that base projections of the future path of debt on a series of estimated relationships that are derived from the same analysis, have limitations to capture all that “soft” information that can be embedded in an expert’s subjective assessment of a country’s economic situation.[4] One example of “soft” information that these types of DSA typically do not contemplate is the possibility that policy makers at a given point in time might be resolute towards making policy changes that break the past-dependence. The expert’s judgment on the credibility of a new policy stance should, in principle, also enter into the analysis. Others examples include: an assessment of the roll-over risks a country faces given the situation in financial markets; the potential effects on debt sustainability coming from the presence of liabilities not recognized by the governments (i.e., “skeletons”); or the potential effects coming from changes in exchange regimes, natural disasters, etc.

[3] See Wyplosz 2007 for a review of this literature.

[4] Moreover, many times there are limitations with data availability that prevent any econometric-based analysis.

In addition, given the uncertainties inherent in doing forecasts in emerging markets, it is very hard to claim that a single model is better than others. As a consequence, the consideration of multiple scenarios is common practice in the day-to-day activity of analysts working in this field. The techniques we explore in this paper are an attempt to make this kind of practice more transparent and systematic.

Nowadays, the use fan charts has become a common practice in the study of risk management in monetary policy and inflation forecasting. From the original application of fan charts implemented by the Bank of England (BoE) in its inflation forecast, there have been many extensions and improvements to the methodology. The Bank of England and other Central Banks have made creative use of this approach in terms of communicating its policies to the market. As future economic variables are affected by today's expectations of what may happen, this communication strategy is seen as important for a Central Bank with an explicit inflation target.

A somewhat similar story may be told regarding debt sustainability. Debt sustainability depends on expectations of future variables and waning confidence in the sustainability of debt in the future may affect many variables today. A fan chart may then also be a useful device for a government to communicate its policy stance to the market. A government that can effectively outline its policies and why they should lead to a low probability of any debt sustainability issue arising may gain significant credibility. Credibility is just as important in debt management as in monetary policy as simply the fear of a lack of sustainability may stifle liquidity and raise interest rates and hence worsening the solvency position. We also suggest that a fan chart analysis may assist in dialogue with the international financial institutions. At times it is useful to suggest that a country's debt is or is not sustainable, but perhaps more constructive are messages about policies that may improve or deteriorate a solvency position. The fan chart analysis allows this type of dialogue and also a way to quantify the effect of certain policy changes that have occurred or that are being contemplated.

What we do in this paper is to discuss techniques that create bridges between the approaches commonly applied to debt sustainability analysis. Our hope is that both policymakers and other end users of DSAs will find this material useful and stimulating enough to encourage them to explore novel approaches to debt sustainability analysis.

1.1 .Background

The fan charts methodology is part of what is known as probabilistic approaches to uncertainty analysis. Uncertainties are characterized by the probabilities associated with events or outcomes of a set of variables. The probability of an outcome can be interpreted in terms of the frequency of occurrence of any of those possible results. Nowadays, the use fan charts has become a common practice in the study of risk management in monetary policy and inflation forecasting. From the original application of fan charts implemented by the Bank of England (BoE) in its inflation forecast, there have been many extensions and improvements to the methodology.[5]

The use of fan charts has been gradually extended to the analysis of debt sustainability. In this paper, we want to critically revisit these efforts. On this issue, Garcia and Rigobon (2004) and Penalver and Thwaites (2006) propose a methodology based on the so-called "risk management perspective." The risk management approach belongs to the group of models that generate a probabilistic distribution based on regression analysis and posterior simulation. Under this framework the risk measure is approximated using a vector autoregressive (VAR) model which captures the correlation pattern between a set of macro variables. Other authors have introduced into the analysis the role of fiscal policy behavior in an attempt to improve the specification of the fiscal behavior in the calculations, and to avoid some undesirable restrictions imposed by the VAR on the specification of the fiscal reaction function.[6]

Other papers incorporate more economic structure into the analysis through the use of theoretical economic models to derive the probability distribution. For example, Mendoza and Oviedo (2006) apply stochastic simulation methods in a dynamic general

[5] The BoE approach was basically based on a subjective calibration of two main parameters of a given probabilistic distribution of inflation. They established a sequential estimation process where authorities interact with the technical team of forecasters in order to achieve a consensus.

[6] In particular, Celasun, Debrum and Ostry (2006) estimates a fiscal reaction function (in the sense of an average fiscal policy) for a panel of emerging market economies. Their methodology also tries to address the common problem of availability of low frequency data in many applications. Other extensions and

equilibrium modeling framework where the co-movement pattern among macro variables is determined by an explicit theoretical structure. Hostland and Karam (2005) combine some elements and ideas from theoretical models jointly with the calibration of some parameters in a more subjective manner.

There has been also a growing literature that focuses on different ways of combining expert subjective judgments, together with the forecasting econometric analysis. Since the pioneering work of Sims (1982) on the use of econometric analysis for policy modeling, there has been an increasing recognition that subjective approaches are a useful complement to purely statistical models, and different proposals have been made in order to connect both dimensions of analysis. The Bayesian framework has been specially encouraged by Sims (see for example Sims 2002 and 2006) in the search for improvements in models of monetary policy.[7]

There is a related stream of literature exploring different approaches to the combination of forecasts.[8] For example, Clemen and Winkler (1999), Leeper and Zha (2003), Adolfson *et al* (2005), and Österholm (2006) address the practical issue of generating conditional forecasting for a given scenario that can be proposed, for

approaches aimed at generating flexible statistical models can be found in the works by Cogley, Morozov and Sargent (2003) and Clements (2005)

[7] In particular, Sims (2002) reviews the problems faced by modelers in central banks, and highlights that important technical development to improve modeling identification can be expected in the near future. The author also addresses the benefits of decentralizing the modeling activity between different teams.

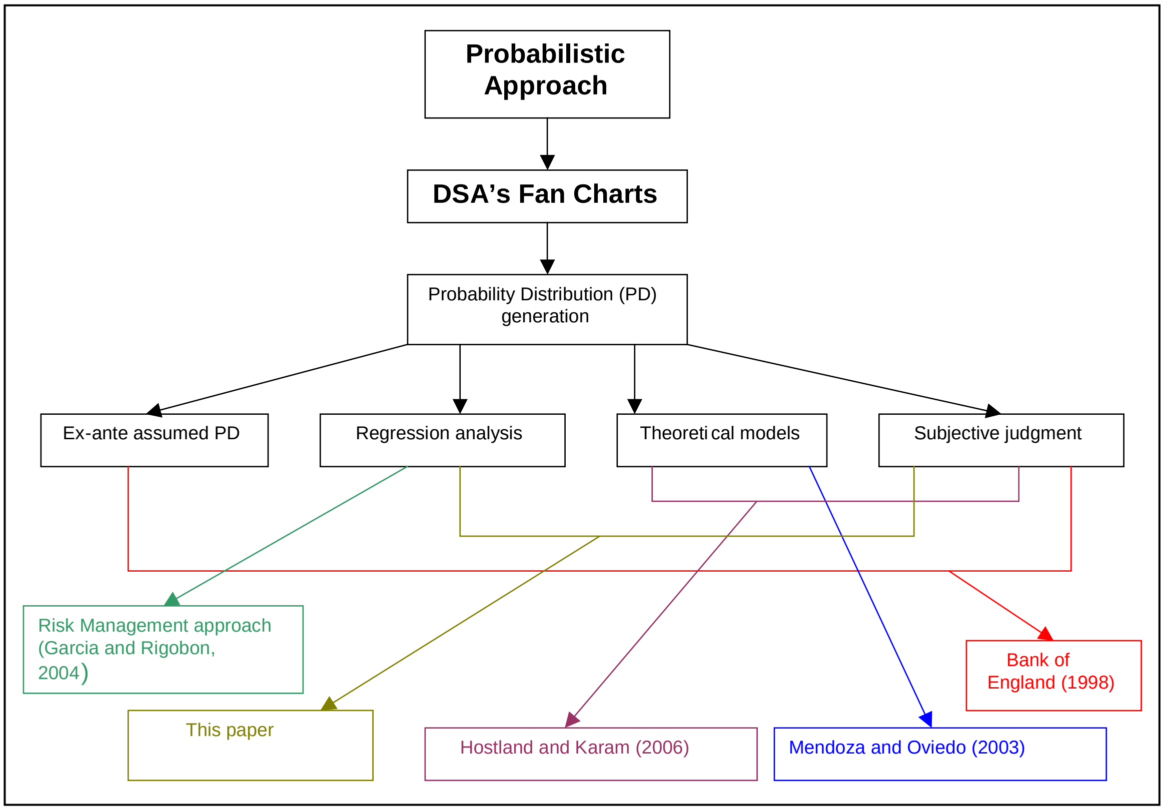
[8] O'Hagan et al (2006) reviews different methodologies in this field.

example, by an expert or a competitive model, but none of these focus on fan charts for DSA (more on this below).

Our main purpose in this paper is to present several alternatives aimed at incorporating subjective measures (i.e., from country analysts, etc.) and/or ad-hoc projections (i.e., from general sources as the International Monetary Fund's World Economic Outlook) into the construction of the fan charts for DSA.[9] The objective is to integrate the econometric approaches with the view or opinion of an external entity in order to generate fan charts that are flexible and also backed by a simple and clear methodology.

Figure 1 shows a synthesis of the main approaches applied to-date to DSA using fan charts analysis and places our contribution in that map. As indicated in the figure, this paper is aimed at proposing methodologies that combine regression analysis and expert subjective judgment in the generation of fan charts for DSA.

Figure 1



The remainder of this paper is organized as follows. In Section 2 we discuss the basics elements involved in the FC methodology applied to the DSA and we propose several methodologies to integrate econometric techniques with external forecasts. In section 3 we summarize and we also enumerate some issues that we think should be considered in the process of designing a methodology for DSA's using fan charts. The final section provides some conclusions.

[9] From now on we will refer to these kinds of projections as "external projections".

2. Fan charts in DSA

2.1 Basics

In this section we briefly describe the main elements involved in the fan charts methodology applied to the debt sustainability analysis. As commented before, the fan charts have been extensively used in inflation forecasting analysis, with the Bank of England as one of the pioneers. But even when the output is similar, many of the concepts and procedures are different.

The starting point is the basic equation of motion for debt as percentage of GDP (see Appendix A for a derivation of this equation from first principles):

$$d_t = \frac{(1+r_t)}{(1+g_t)} d_{t-1} - f_t \qquad (1)$$

Where d is the debt to GDP ratio, r is the interest rate, g is the annual growth rate of GDP, and f is the fiscal primary surplus as percentage of GDP and t indexes time.[10] Equation (1) can be expanded in several dimensions, depending on the characteristics of the economy under study. However, for expositional simplicity we will focus here on the simplest case.

[10] The primary fiscal surplus is the difference between total government revenues and expenditures net of interest payments on the outstanding debt.

Any DSAs involve making projections about possible future paths of the debt-to-GDP ratio (d_t) over a given forecasting horizon. In order to make these projections it is necessary to forecast the possible paths to be followed by the variables entering into equation (1). Traditional DSA approaches rely on medium-term projections (i.e., 3 to 5 years) of the variables that enter (1) to assess a baseline scenario for d_t.[11] Instead, uncertainty-based approaches consider the variables that enter (1) to be stochastic and seek to determine their probability distribution in order to come up with a range of possible outcomes over the forecasting horizon. For the purpose of generating the probability distributions, there are several options. For example, the so-called risk management approach (i.e., Garcia and Rigobon, 2004) fits into the group of methodologies that use regression analysis with posterior simulation in order to generate a range of possible outcomes for the variables that enter (1).[12] The key point is that in order to incorporate uncertainty into the analysis, what is needed it to come up with multiple projections for the possible realizations of d_t over the forecasting horizon, and this in turn requires simulation techniques.

In general, the construction of a fan chart involves the following sequence of steps:

1) Define the equation of motion of the debt (i.e., equation 1).
2) Select a time-horizon (T) and a periodicity (i.e., monthly, quarterly, annual) to estimate the model and construct the fan chart projections.

[11] See Wyplosz (2007) for a review.
[12] Other options are the ones presented in Figure 1.

3) Collect the relevant data.

4) Determine a methodology for the projection of the variables that influence the dynamics of the debt.

5) Use simulation to come up with a range of possible outcomes for d_t

In the next subsections we critically revisit different options available for performing (4) and (5) which are the core of the methodology.

2.2 Methodology 1: Risk Management Approach

The procedure followed by Garcia and Rigobon (2004), Penalver and Thwaites (2006), and Celasun, Debrun and Ostry (2006) is to estimate the conditional means and variances for the variables that enter equation (1) from the data and use them to simulate the multiple future paths of the debt-to-GDP ratio. In the three cited papers, the estimation method used is a vector autoregression (VAR).

The VAR provides two of the main elements of the stochastic simulation. First, the variance-covariance matrix of the residuals characterizes the joint statistical properties of the contemporaneous disturbances affecting the debt dynamics. Second, the estimated coefficients of the VAR allow to make projections of the variables that enter into the equation of motion of the debt, and thus to generate a central trend as well as alternative paths.

The Vector Autoregressive Model (VAR) [13] is popular due to its predictive power and because it does not rely on the construction of a theoretical model. Consider the following autoregressive model:

$$Y_t = \mu_0 + \sum_{k=1}^{p} \mu_k Y_{t-k} + \xi_t \quad (2) \quad where\ Y_t = (r_t, g_t, f_t, \pi_t, e_t);\ \ \xi \sim N(0,\Omega)$$

In this case Y_t is a vector that contains N variables (N-VAR). For instance, $Y_t = (r_t, g_t, f_t, \pi_t, e_t)$ represents the interest rate, *g* is the GDP growth rate, *f* is the general government primary balance, π is the inflation rate and e is the exchange rate, and where *t* indexes the period and *k* the order of auto-regression. ξ is the error vector with variance-covariance matrix Ω , and μ_k is a vector of coefficients. Note that the variables included in the VAR are more than the variables that enter into the equation (1). This suggests that it is possible to include into the econometric specification other variables that may not enter in the equation of motion of the debt, but that may influence the evolution of the first ones.

The next step is to simulate multiple random shocks (M) for each of the variables included in the VAR using Monte Carlo simulation techniques, thus generating T vectors of the form [M x N] such that:

$$x_\tau^s = x_\tau^{VAR} + \eta_\tau \quad (3)$$

$$\eta_{t+1},...,\eta_T \ \ for\ \tau \in [t+1, T]$$

$$\eta_\tau = W\upsilon_\tau \ \ where \quad \upsilon_\tau \sim N(0,1)$$

Where W is obtained through the Cholesky decomposition of Ω and hence represents the joint statistical properties of the variables entering the VAR. Note that x_t is the subset of variables in vector Y_t that enter equation (1).

In this approach, the point estimates coming from the VAR constitute the central tendency of the forecast for each of the variables included in the simulation. Each of the simulated paths (x_τ^s) is the result of the sum of the point estimate coming from the VAR (x_τ^{VAR}) and one of the generated errors (η_τ). As a consequence, the dispersion of the forecasts of the fan chart is determined by Ω, the variance-covariance matrix of the residuals of the VAR. Note that the method is not sensitive to the order of the variables in the VAR as we are not capturing here causal relationships, only the joint dynamics for all the variables. Replacing the results of the simulations in the equation of motion of the debt (1) we obtain M paths for the debt to GDP ratio.

Next, the fan chart is constructed for the forecasting horizon of choice (i.e., T=5 years) repeating the exercise, generating M new shocks for each period. As ways of illustration,

[13] See Sims (1980).

we have constructed fan charts using this methodology using data from Colombia, Brazil, Mexico, Peru and Uruguay. For concreteness we set (M=1000) and run a VAR (1) to the variables included in Y_t in equation (2). The resulting fan charts are in panel (a) of Figures 2, 3, 4, 5 and 6 (Appendix B). Each plot also includes the standard deviation of the forecast in the left scale. We use annual data from the IMF's World Economic Outlook (WEO).[14] It is interesting to note that, because the simulations for the future paths of d_t are based on replicating forward the past pattern of correlations among the relevant variables, the direction of the resulting fans is somewhat past dependence (i.e., if the debt-to-GDP ratio was falling prior to the forecasting horizon, it is likely to continue to fall for the forecasting horizon).[15]

One of the main advantages of the risk management approach is that it is simple and relatively easy to implement. Others authors try to incorporate more economic

[14] In these examples: economic growth corresponds to the percentage annual variation in the GDP, in nominal prices of national currency (WEO code: WNGDP); Inflation rate refers to the percentage annual change in the Consumer Price Index (WEO code: WPI); Exchange rate, expressed in national currency per U.S. Dollar, is used to calculate its annual percentage variation (i.e., depreciation rate) (WEO code: WDI); As a proxy of the actual interest rate paid, we take the ratio of total debt interest paid (WEO code: WDSI) in current US dollars, to total debt outstanding at year-end, in current US dollars. Fiscal primary surplus as percentage of GDP is obtained dividing general government primary balance in current national currency, by the GDP, in current national currency. Finally to account for the ratio of debt as a percentage of GDP, we divide the total debt outstanding at year-end by GDP. We construct the total debt series using data on domestic debt from Jeanne and Guscina (2006) database, plus public and publicly guaranteed external debt published by the World Bank's World Development Indicators (WDI) database.

[15] Some caveats related to the data should be done. First, we use WEO data for our simulation and thus, our choice of periodicity (step 2 above) is annual. This might be a problem as higher frequency data is usually preferable for VAR analysis (see Garcia and Rigobon, 2004). Nevertheless, as discussed in Celasun et al (2006), budgetary data (which is indispensable in DSAs) is typically scarce and it is available with an annual frequency only. Another issue refers to the fact that using WEO data we can only estimate the interest rate paid on the external portion of the total public debt. We use this as a proxy for the interest rate on all debt payments, irrespective of the nature of the debt. While these are all certainly important issues that should be considered carefully when implementing this approach to DSAs, our contribution here is to discuss the methodological issues, which are not invalidated by data limitations.

structure into the analysis through the consideration of the role of the fiscal policy behavior (Celasun et al 2006), through the use of other risk analysis techniques (for example, Barnhill and Kopits, 2003), or in general trying to combine some features from more formal economic models jointly with the calibration of a set of parameters (for example, Hostland and Karam 2006). However, all these methods share the characteristic that they rely exclusively on statistical techniques. As such, projections about the evolution of the debt-to-GDP ratio are entirely based on past performance. Thus, they may be less useful to analyze issues such as what is the likely evolution of the debt-to-GDP ratio if the authorities are resolute in following some fiscal targets that may be different from those in the past.

Even when VAR models have been widely used in the field, there are other potential methodologies that can be also explored. Structural VARs might be used to interpret the effects of different economic policies (i.e., the interest rate can be consider as an exogenous variable in this context) and/or to incorporate some restrictions on the relations between the macroeconomic variables that can arise from theory or from judgment (for instance, some stylized facts that occur during stress situations can be considered). Bayesian techniques and more specifically Bayesian VARs are very useful to incorporate experts' beliefs and to compare and combine models. The Bayesian framework has been encouraged by Sims (2006) in the context of monetary policy models. Different techniques may be also explored for a model-consistent incorporation of judgment or general external projections (see Österholm, 2006). These techniques are usually relatively easy to implement and fit a Bayesian framework.

In our view, the application of stochastic simulations into the analysis of debt sustainability, and more specifically the econometric-based methodologies, can be enhanced with the consideration of subjective measures that could come from the judgment of experts or external forecasts (i.e., as projections form the IMF's World Economic Outlook). These issues are discussed in the next subsection.

2.3 Alternative methodologies

2.3.1. Methodology 2: External forecasts.

One problem with the VAR approach is that it is purely backward looking. It hardly will be able to take into account shifts in policies and ongoing structural changes in the economy. The more frequent these situations, the less reliable the econometric estimates for the prediction of future evolutions of the variables of interest. Another problem is that it is data intensive and in many developing countries the availability of long time series for relevant macroeconomic variables (which are indispensable for a full-fledged econometric-based analysis) is limited.

One simple alternative to the VAR approach is given by the method advanced in Borensztein, Cavallo and Valenzuela (2009). In this approach, the projections of each of the variables included into the debt equation (1) are made according to the following equation:

$$x_{\tau}^{s} = x_{\tau}^{Ext} + e_{\tau} \; ; \; for \; \tau \in [t+1, T] \quad (4)$$

$$where: \; e \sim N(0, \hat{\sigma}^2)$$

Where x_{τ}^{Ext} --i.e., recall that x_t is the set of variables that enter equation (1)—is the external projection provided by the expert (or any other external source such as the IMF's WEO forecasts, or countries' official projections); e_t is a vector [M x N] of simulated errors which have variance σ^2 equal to the sample variance of each of the series. Under this specification the simulated errors are not correlated and no additional information from the time series is extracted other than the sample volatility. The central trend of the simulation is given by the x_{τ}^{Ext} values (i.e., the WEO forecast). Notice that according to equation (4), when x_{τ}^{Ext} remains as a constant, each of the components of the vector evolves according to a random walk with a drift. This model is akin to the standard DSA analyses (with the central trend given by some benchmark projection), but improves upon it by introducing uncertainty into the analysis. Moreover, because no regressions are run, it is less demanding in terms of data. The drawback is that the uncertainty is entirely based on the historic volatility of the corresponding series and the correlation between the variables is ignored.

We use the same data from Brazil, Colombia, Peru, Mexico and Uruguay to build the fan charts using this methodology. The central projections in each case are built based on the corresponding WEO forecasts for each of the variables that enter the equation of motion of the debt for the period of the forecast. The results presented in panel (b) of Figures 2, 3, 4, 5 and 6 (Appendix B). Note from the figures that the past dependence is now broken: the baseline trajectory in this simulation is now based on external projections about the most likely scenario for each of the relevant variables that drive the debt-dynamics. The “fan” around the baseline trajectory incorporates the risk associated to that forecasts, where risks are assessed through simulations of probable future debt paths based on observing the historical variance of these variables. Note, however, that the amplitude of the fan is usually smaller than the fans in panel (a) –i.e., the standard deviations of the forecast on the left hand-side scale are usually smaller—. The reason is that, unlike the fans generated through the VAR and reported in panels (a), the fans in panels (b) ignore the pattern of correlation between the relevant variables and use only their historical variances. And given that the pattern of correlation between variables in developing countries (as the ones included in these examples) is usually destabilizing of debt sustainability,[16] ignoring it reduces the assessment of risk.

In summary, one advantage of this method is that it provides an easy way of incorporating the subjective beliefs of the analyst (in this example, the WEO forecast) into the projections and the simulations do not require the computation of a VAR which may be unreliable for many countries with limited data and a pattern of structural brakes in the data. Nevertheless, the simulated errors are not correlated which could generate trajectories that may not be realistic. For instance, the methodology may find as feasible some co-movements of the variables that are not likely to happen, at least taking into account the historic evolution of the economy (i.e., a positive correlation between output and the nominal exchange rate in developing countries). The next methodology seeks to improve upon this one by explicitly incorporating the co-movement of the variables into the forecasts.

2.3.2. Methodology 3: External forecast with correlated errors.

The simplest way to improve upon methodology 2 is to incorporate correlated errors in the simulation, where the correlation structure comes from a VAR. In this case we use

[16] For example, recessions deteriorate fiscal accounts even without discretionary expansionary fiscal policies, increases the real interest rate, induce inflation and depreciate the exchange rate all of which affect adversely the sustainability of debt indexed to foreign currencies.

the following equation for the projection of the variables included in the equation of motion of the debt:

$$x_\tau^s = x_\tau^{Ext} + \eta_\tau \; ; \; for \; \tau \in [t+1, T] \quad (5)$$

$$where: \; \eta \sim N(0, \hat{\Omega})$$

Where now x_τ^{Ext} is a vector of external projections for the variables included in the debt equation (same as methodology 2), η_t is a vector [M x N] of simulated errors with variance $\hat{\Omega}$, that equals the variance-covariance matrix of the VAR residuals. Thus, a VAR is estimated like in the risk management approach, but instead of using the coefficients of the VAR to obtain the central projection for the debt-to-GDP ratio, the central projection is given by some external forecast, hence the VAR is used only to obtain a correlation matrix for the joint distribution of the errors.

Once again we use the data from the same four countries to compute the fan charts using this methodology. The results are illustrated in panel c of Figures 2, 3, 4, 5 and 6 (Appendix B). The main difference with the fans that were generated through the previous methodology (i.e., panel b) is that the cone of uncertainty around the baseline projections (which is based on external forecast) is now derived from simulations based on the historical correlations between the relevant variables. Moreover, because the VAR can include more variables that the ones that enter into the equation of motion of debt (i.e., equation 1), the set of “relevant variables” may now be expanded to include those variables that may affect the debt dynamics indirectly through the impact of those variables that enter (1). The drawback, however, is that these estimates are only as good as the estimates of the VAR which may turn out to be unreliable (or even incomputable) for several countries.[17]

It is important to note that, while we use the variance-covariance matrix that results from computing different VARs using the data of each of the four countries, in principle, the variance-covariance matrix could also be the product of a structural vector autoregressive (SVAR) estimation. In that case, an explicit theoretical relationship between the variables is imposed. This approach of “manipulating” the variance-covariance matrix can also be extended to specify different matrices to be used depending on the economic scenario. For example, it can be stated that the relationship between macro variables can be very unstable, especially for emerging markets economies, and hence this discontinuity should be taken into account in the analysis. As discussed before, during periods of crises, developing economies experience co-movements in their macro variables that actually deteriorate debt sustainability: i.e., recession, fall in revenues, worst fiscal results, hike in interest rates, inflation and exchange rate depreciation. On the other hand among industrialized economies, an automatic stabilization process may take place during crises, whereby lower output

[17] Note that this is problem shared with the risk management approach which also uses the estimates from a VAR to assess the pattern of correlation among the variables.

growth is accompanied by expansionary monetary or fiscal policies that induce a fall in interest rates and consequently improve debt sustainability. It could be the case that in tranquil times emerging markets behave more like the developed countries, and conversely during crisis experience the vicious circle described above. Therefore, different variance-covariance matrix could be considered in the DSA.

2.3.3 Methodology 4: Weighted projections.

The next approach combines methodologies (1) and (3). Consider the following model:

$$x_\tau^s = \beta * x_\tau^{Ext} + (1-\beta) * x_\tau^{VAR} + \eta_\tau \; ; \; for \; \tau \in [t+1, T]$$
$$where: \; \eta \sim N(0, \hat{\Omega}) \qquad (6)$$

This method explicitly combines the external forecast (x^{Ext}) with the projection generated through a VAR (x^{VAR}), and assigns weight β ε [0,1] to the latter and weight (1-β) to the former. Note that β can also be a vector of weights, with different values for different variables and/or years of the simulation. Note that when β=0, we obtain the same results as in methodology (1). If, instead β=1, we replicate methodology (3). As with the previous methodology, η_t is a vector [M x N] of simulated errors with variance $\hat{\Omega}$, that equals the variance-covariance matrix of the VAR residuals.

The fan charts generated through this methodology (using a uniform weight β=0.5) are plotted in panel (d) Figures 2, 3, 4, 5 and 6 (Appendix B). What this method is doing is to enable the forecaster to put some weight on baseline projection that comes from external forecast and some weight to the baseline projection that comes from the predictions of the VAR. However, the choice of the weights is not straightforward.

One possibility is to use ad-hoc weights (i.e., β=0.5 or any other weight that the analyst considers as reasonable given her information set). This may be a reasonable pragmatic approach in contexts where it is hard to integrate many of the elements influencing the future fiscal stance into a single and well specified reduced-form econometric model and/or in a structural model. The combination of different set of forecasts may in turn, be a useful approach to study how projections change if different weights are assigned to the econometric-based forecasts and the external projections respectively. As more information becomes available, this approach may help to calibrate which combination of forecasts produce better outcomes in different times and circumstances; or even study if there is some kind of convergence between the forecasted densities and if the econometric models can be improved by incorporating some of the features that the external forecasts seem to be capturing.

As a way of practical guideline for selecting the weights in this methodology we suggest to start with a baseline assumption (typically equally weighted forecasts) and then consider the magnitude of the changes in the resulting fan charts to different weights (including the extremes cases 0-1 which correspond to the standalone methodologies 1 and 3). The closer inspection of the endogenous projections generated by the econometric model and the exogenous external projections can shed light on the individual factors driving the changes in the projected densities (i.e., a credible change in the official fiscal policy that is not fully captured by the regressions; or a change in the exchange rate regime, a natural disaster, etc).

As it was commented in the background section, there is a related stream of literature exploring different approaches for the combination of subjective judgments (i.e. judgment-based forecasts), together with econometric models. These subjective judgments are usually summarized in the form of experts' probability distribution in risk analysis. For example, Clemen and Winkler (1999) discuss a variety of more formal forecast combination methods, commenting conceptual and practical issues related with mathematical and behavioral methods of aggregation. As the authors notice, experts' analysis can provide valuable information, particularly when there is limited availability or access to "hard" data. But, one important point also emphasized is that there is no single, all-purpose combination rule for different sources of judgments and projections. As a consequence, even though simple combination rules (e.g. a weighted average) provide helpful benchmarks for the analyst, the selection of the best combination strategy should take into account all the particular characteristics of each situation.

Another related contribution is made by Österholm (2006). He discusses different approaches for the combination of judgments into a density projection econometric model for forecasting individual macroeconomic variables (GDP growth, inflation and interest rate). In a nutshell Österholm suggest to model the different judgments as a scenario in general-equilibrium type macroeconomic models. Then the predictive densities associated with the different scenarios can be combined into one final fan chart via a weighted linear combination (i.e. a linear opinion pool). Our approach in the combination of forecasts resembles this methodology. Nevertheless whereas Österholm proposes a combination of the econometric model's (a Bayesian VAR) endogenous forecast with different alternative scenarios for the set of forecasted variables, we are proposing a combination of the ex-post forecasts from the VAR model together with the external set of forecasts (it would be analogous to the combination of methodology 1: the risk management approach and methodology 3: external forecast with correlated errors). The resulting weighting densities are not easy to characterize in either case, suggesting the use of numerical methods to evaluate the comparative performance of both approaches.

2.3.4 Ex-Post assessment of the different methodologies

While it is very hard to assess which methodology yields the most accurate results, as time passes and more data becomes available interested analysts may do some retrospective tests. There are two relatively simple methods that may be applied for such tests. However, in both cases we need to simulate fan charts for previous years so it is necessary to have the required data, in particular, the data on external projections for past years in order to evaluate the performance of the methodologies that use these projections.[18] The first method consists in comparing one or more of the risk measures that can be calculated from the fan charts (i.e., the probability of exceeding a given debt-to-GDP threshold in a 5 years horizon), with a proxy variable assessing the riskiness of sovereign debt assigned by the market (i.e., the EMBI spreads).[19] Then, simple panel regressions allow to quantify how much of the variability in the spreads (i.e., country risk) is explained by the estimated risk measure for each of the methodologies and of course for different values of the weights in the case of methodology 4. Being able to replicate the analysis for previous years increases the data points available for the regressions.

A second possibility is to do a retrospective test of the performance of the different fan charts in terms of their ex-post accuracy. The fan charts can be simulated for past years and a measure of the distance between the forecasted densities in each year and the observed debt-to-GDP ratios can be computed. During the process it would be desirable to consider the accuracy of the central projections as well as the standard deviations of the simulated densities for each of the years in the sample.

[18] For example, if the fan charts were produced using data on external projections available through the IMF World Economic Outlook (WEO), it may be useful to have access to past issues of the WEO containing the old projections in order to conduct the retrospective analysis.

3. Taking Stock

To summarize, the main elements of the approaches proposed in this paper to use fan charts for DSA analysis are as follows:

(1) VAR Approach: select an equation of motion of debt. The variables that enter the equation are stochastic and correlated. Assume historical correlations – computed through suitable regression analysis (e.g. a VAR) — are likely to be relevant in the future. The baseline scenario for the selected forecasting horizon is the central projection arising from the VAR. Randomly generate a large number of shocks to all variables and associate a given debt path with each shock –following the selected equation of motion of debt—, much as standard DSA (i.e., the DSA that only rely on medium term projections of the relevant variables combined with stress tests) except that each debt path now comes with a probability of occurrence. Figure 7 in the Appendix D summarizes the step-by-step computational process involved in generating fan charts for DSA using a VAR.

(2) External Projections: select an equation of motion of debt and use external forecasts for the variables that enter into it to trace a baseline of the resulting

[19] This is similar to the strategy followed by Garcia and Rigobon 2004, but as we have limited historical data on the external projections we propose to use a panel regression model to assess the performance of each methodology.

evolution of d_t for the selected forecasting horizon. Assume that historical variances of the variables that affect the evolution of the debt are likely to be relevant in the future. Randomly generate a large number of shocks to all variables and associate a given debt path with each shock –following the selected equation of motion of debt—, much as in the standard DSA, except that each debt path now comes with a probability of occurrence.

(3) <u>External Projections with correlated errors</u>: select an equation of motion of debt and use external forecasts for the variables that enter into it to trace a baseline of the resulting evolution of d_t for the selected forecasting horizon. Assume historical correlations – coming from the variance-covariance matrix estimated with a multivariate time-series regression model —are likely to be relevant in the future. Randomly generate a large number of shocks to all variables and associate a given debt path with each shock –following the selected equation of motion of debt—, much as in the standard DSA except that each debt path now comes with a probability of occurrence.

(4) <u>Weighted average of projections</u>: select an equation of motion of debt. For the baseline scenario, combine the forecast coming from the VAR in (1) and the external projections used in (2) and (3) using ad-hoc weights or a different approach. Assume historical correlations –computed through suitable

regression analysis—are likely to be relevant in the future. Randomly generate a large number of shocks to all variables and associate a given debt path with each shock –following the selected equation of motion of debt—, much as in the standard DSA except that each debt path now comes with a probability of occurrence.

Table 1 in Appendix C summarizes what in our view are the main advantages and disadvantages of each of the proposed approaches. While the list of clearly not exhaustive, it provides some information that we think is useful to the analyst in helping her pick the best approach for her needs. Our general assessment based on the information summarized in this table, is that the combination of a VAR approach in order to characterize the uncertainty prevailing in any economic environment, with expert judgment is a promising avenue for analysis.

What type of questions can the use of fan charts for DSA enable practitioners to answer? Because fan charts are just the accumulation of multiple probable paths for the future realizations of the debt-to-GDP ratio, one question that they can help to answer is the following: what is the probability that by the end of the chosen forecasting period the debt-to-GDP ratio may be above a given threshold? The answer is simply the number of simulated paths that lie above the threshold over the total number of simulated paths. As suggested in the previous section these probabilities in turn can then be compared to observed market risk assessments (i.e., credit ratings or sovereign bond spreads) to evaluate the predictive capacity of the fan charts. In addition, different measures of dispersion and skewness can provide useful information on the risk involved in the projected period.

4. Conclusion

Debt sustainability analysis is at the centerpiece of macroeconomic financial programming. It is an essential task for policy makers that need to make decisions about budgeting or about strategies on how to finance public expenditures. At the same time, it is a necessary tool for creditors in assessing the risks involved in lending operations.

For international multilateral institutions that have a mandate to promote economic development, it is also important to have a thorough assessment of a member country debt situation and forecasts before approving a lending program. Experience shows that lending in situations where solvency is at risk is a serious threat for the country, even if the institution’s capital is protected by the preferred creditor status. It is well established in the literature that debt crises are associated with big economic, social and political costs that derail economic and social progress. Thus, getting the lending strategy right is crucial to accomplish their mission.

In this paper we propose techniques that create bridges between the deterministic and the uncertainty-based approaches for DSA. Our hope is that both policymakers and other end users of DSAs will find this material useful and stimulating enough to encourage them to explore novel approaches to sustainability analysis. However, it is important to always remember that all approaches to DSA have to rely on assumptions about the future evolution of key macroeconomic variables. Therefore, as suggested by Wyplosz (2007), the usefulness of the conclusions is directly related to the validity of these assumptions, which by definition are neither safe nor testable. The same is true of the data: the quality of the data input is crucial. In any event, there is no substitute for the researcher's good judgment in selecting the appropriate data sources and the most appropriate assumptions applicable to the country under study.

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Appendices:

A- Equation of motion of debt

Let Dt, Gt and Ft be the public debt, the GDP and the Fiscal Surplus in time t. The equation of motion of debt is defined as:

$$D_t = (1+r) * D_{t-1} - F_t$$

Dividing both sides by Gt we obtain:

$$\frac{D_t}{G_t} = (1+r_t) * \frac{D_{t-1}}{G_t} - \frac{F_t}{G_t}$$

$$d_t = (1+r_t) * \frac{D_{t-1}}{G_t} * \frac{\frac{1}{G_{t-1}}}{\frac{1}{G_{t-1}}} - f_t$$

$$d_t = (1+r_t) * \frac{D_{t-1}}{G_{t-1}} * \frac{1}{\frac{G_t}{G_{t-1}}}$$

$$d_t = \frac{(1+r_t)}{(1+g_t)} d_{t-1} - f_t$$

B- Fan Chart Figures

Figure 2: Brazil.

Panel (a)

Panel (b)

Panel (c)

Panel (d)

Figure 3: Colombia.

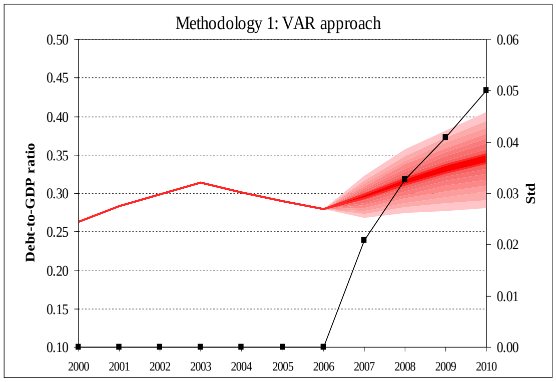
Panel (a)

Panel (b)

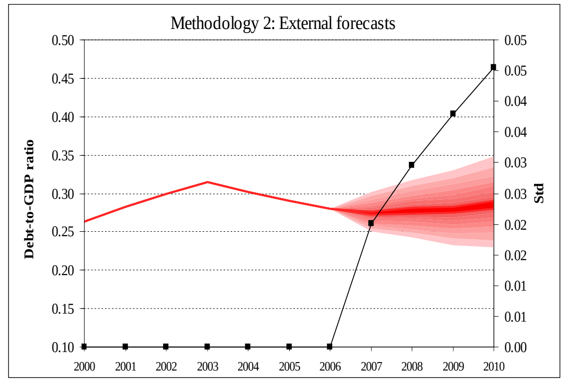
Panel (c)

Panel (d)

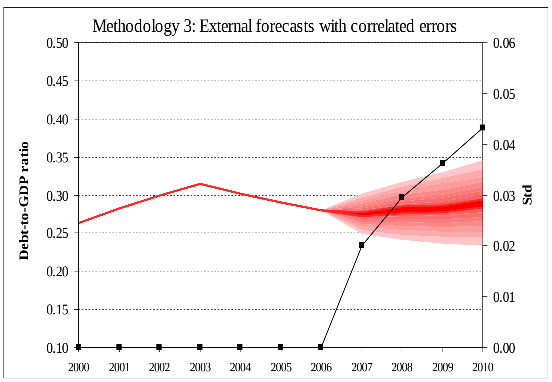
<u>Figure 4:</u> Mexico.



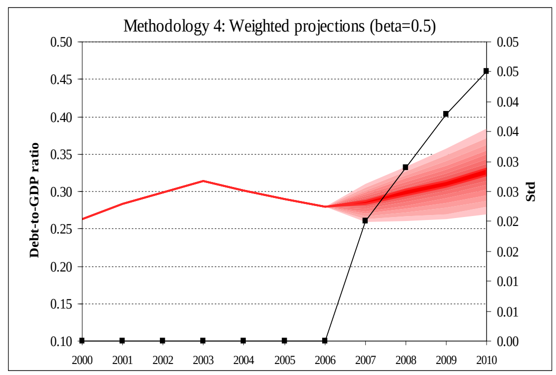
Panel (a)



Panel (b)

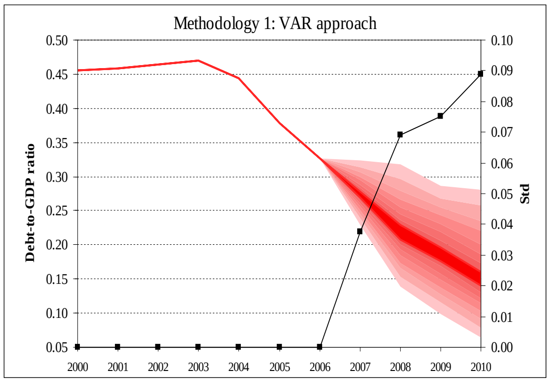


Panel (c)

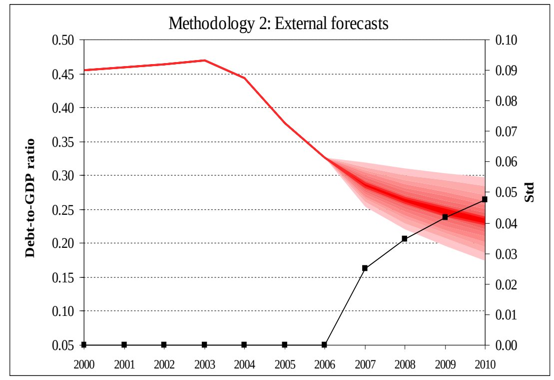


Panel (d)

Figure 5: Peru.



Panel (a)



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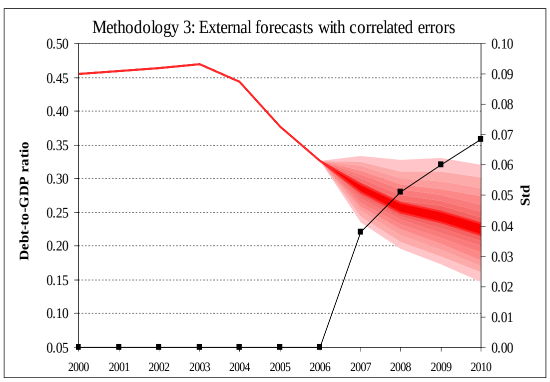
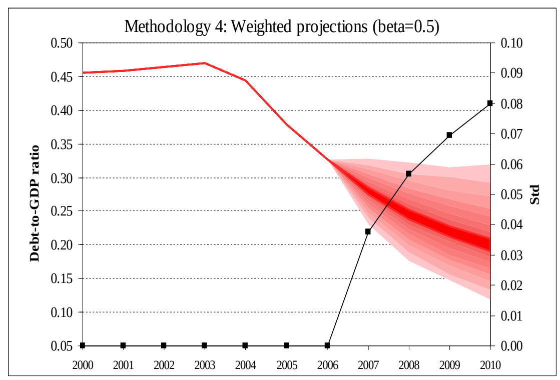
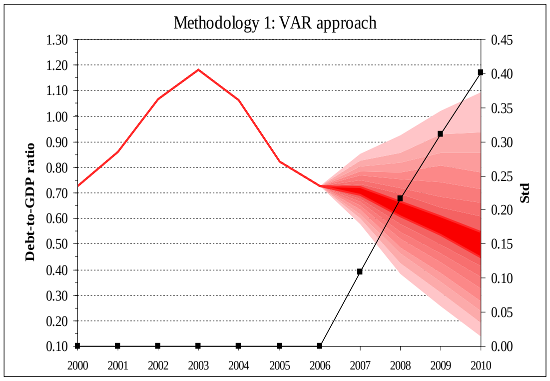


Panel (c)

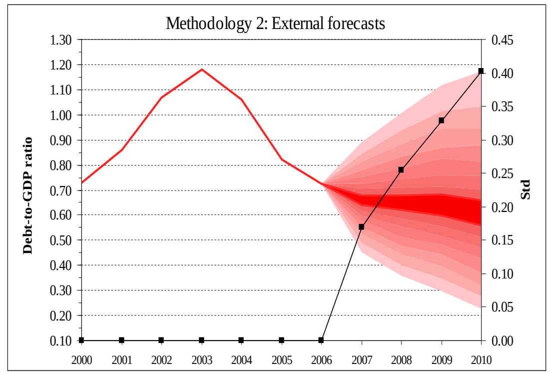


Panel (d)

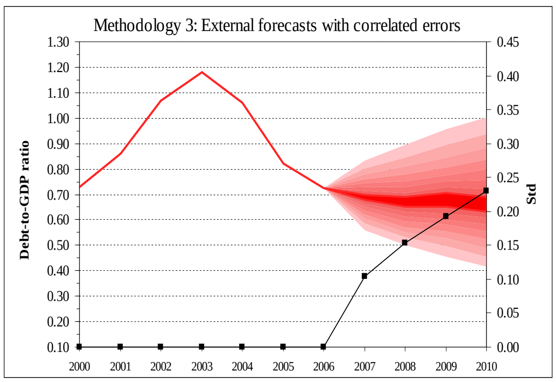
Figure 6: Uruguay.



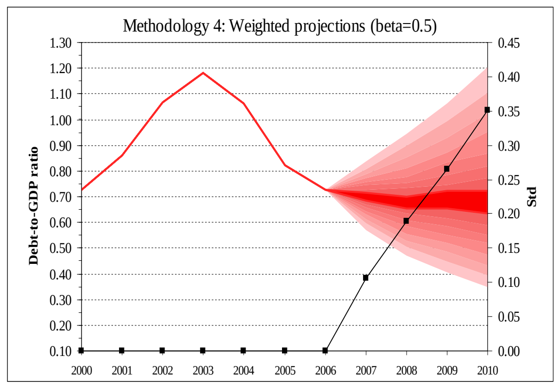
Panel (a)



Panel (b)



Panel (c)



Panel (d)

C- Table 1: Advantages and Disadvantages of the different approaches

Table 1 summarizes what in our view are the main advantages and disadvantages of each of the proposed approaches. Within each category, we offer a “+” classification for those aspects of the methodology that we consider to be advantages, “-“ for disadvantages, and “?” for unknowns.

Approach	+/-/?	Observation
VAR	+	1. It better captures the structural trends present in the data
		2. It allows for improvements on the baseline econometric estimation technique (for example, with the use of Structural and Bayesian VARs).
		3. There are more and better known measures of performance for the baseline econometric estimation models.
		4. VAR models are flexible and usually have a better fit than alternative approaches.
	-	5. As the VAR model is purely backward-looking, it is not equipped to take into account structural brakes in policies or other structural changes.
		6. Uncertainty and instability in the standard models: the unrestricted VAR approach and the estimated var-cov matrix can be unstable, particularly when applied to emerging economies. Dynamics are different in tranquil times and bad times.
		7. Usually the unrestricted VAR models demand higher-than-annual frequency data (typically quarterly data). Budgetary data of such frequency are often either unavailable or unreliable for the purpose of policy evaluation (Celasun et al., 2006).
	?	8. The central trend is completely determined by the econometric model.
External projections	+	1. Forward-looking. Allows the researcher to incorporate possible structural brakes or policy shifts immediately.
		2. Easy implementation. Does not require estimating a VAR.
		3. Flexible introduction of additional shocks (e.g., natural disasters in Borensztein et al. 2009).
	-	4. It is less sensitive to structural trends than econometric models.
		5. Very sensitive to the historical volatility of the series. General problem in developing countries: to include or not to include crisis years?
		6. The uncertainty introduced with the simulation is only attached to empirical facts through the historic volatility of the series. There is no recognition of interactions or correlations among the variables that go into the debt equation (e.g., in many developing countries the lack of automatic stabilizers could be highly relevant).
	?	7. The central tendency is completely driven by external projections. It would be positive or negative depending on the relative importance of structural trends vs. structural brakes.
		8. The whole focus mainly relies on the external projection.
External projections with correlated errors	+	1. Has all the positive elements of the external projections approach, plus the additional advantage of the introduction of correlated errors.
	-	2. It is less sensitive to structural trends than econometric models are.
		3. Var-cov matrix could be very unstable during periods of crisis.
Weighted VAR	+	1. In general, this approach combines some of the strengths of econometric-based techniques and of techniques based on external projections. Many of the weak points in the VAR approach are solved by external projections with correlated errors, and vice versa.
	+	2. The importance of a model-consistent consideration of expert subjective beliefs is generally recognized. (See, for example, the discussion of Sims (2002) on the BoE's FC generation process.)
	-	4. Ad hoc selection of the weighted parameter. There are some alternatives in the literature that can be explored; see, for example, Clemen and Winkler (1999) and Österholm (2006).
	-	5. It is a less standard approach, so new testing and evaluations tools have to be developed.

Figure 7: Summary of the process of generating fan charts for DSA using VAR.

1

Estimate a VAR model (for ex. in Eviews), of the form

$$Y_t = \gamma_0 + \sum_{k=1}^{p} \gamma_k Y_{t-k} + \xi_t$$

In our case: VAR(1), Yt={nominal domestic interest rate, nominal foreign interest rate, nominal exchange depreciation, real gdp growth rate, inflation[gdp deflator, and primary surplus-to-gdp ratio]}, and:

$$\xi_t \sim N(0,\Omega)$$

This vector is the vector of "risk inputs" that then will feed the debt motion equation

2

What we need from the VAR:

1) Ω: Var-Cov matrix
2) VAR Model

3

Simulations of risk inputs:

We need to generate errors for the projection time window [t+1, T]. But the simulated errors should respect the correlation pattern given by Ω. So we use a Choleski factorization:

$$\hat{\xi}_{t+1}, \ldots, \hat{\xi}_T$$

$$\forall \tau \in [t+1, T], \quad \hat{\xi}_\tau = W \upsilon_\tau$$

$$\upsilon_\tau \sim N(0,1)$$

Where W is the Choleski factorization of Ω, i.e.:

$$\Omega = W'W$$

4

So, the VAR generates forecast of Y consistent with the simulated shocks.

As shocks occur each period, the VAR produces joint dynamics responses of the elements of Y.

The method is not sensitive to the order of the variables (we are not capturing here causal relationships, but joint dynamics for ALL the variables).

From 1 it can be seen than to project Y from t+1 to T, it is needed the VAR model (the regressions coefficients + the intercepts) + the errors (the simulated errors conserve the same correlation pattern given by Ω)

5

Using the vector Y from t+1 to T (projection time window) in the motion equation of debt:

[insert debt equation]

A set of simulated paths is obtained (a vector of t+1 to T columns by the number of simulations)

6

From the vector of simulated trajectories for the debt-to-gdp ratio, the corresponding percentiles are taken in each year and the Fan Charts graph is produced.

Different risk measures can be obtained: Prob. of exceeding given thresholds (cf. VaR), std. dev, central trend, skewness, etc.